

# Propagation of Geometric Structures in Smooth Hybrid Systems

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## Abstract

This talk will be concerned with the preservation and propagation of differential geometric structures in state-triggered hybrid dynamical systems. The first part deals with the evolution of observables (Koopman) and of uncertainty densities (Frobenius-Perron). These two trajectories are dual to one another as the former evolves functions while the latter evolves volumes. Asymptotic convergence of densities provide information on ergodic and statistical properties of the underlying hybrid system.

Next, variational impulsive systems are presented. These systems necessarily preserve both the energy and the symplectic structure across events. An important class of these systems arise in optimal control where the maximum principle asserts that, under sufficient regularity, the evolution of the co-states preserve these two structures. Moreover, these lifted dynamics admit a trivial solution to the corresponding Frobenius-Perron operator.

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