Categorical Lyapunov Theory: From Hybrid Systems to Coalgebras... and Back

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Abstract

The hallmark of hybrid systems is the combination of discrete and continuous behavior-and it is the interplay of these two system types that makes analyzing hybrid systems challenging. To mitigate this complexity and lay the foundations for a general theory of open hybrid systems, we model systems categorically as coalgebras, a representation that uniformly captures continuous, discrete, and hybrid dynamics. We then generalize stability via categorical Lyapunov Theory (CLT). In this setting, stability is certified it via Lyapunov morphisms using the single inequality: $F(V) \cdot f \leq \sigma \cdot V$. This yields a categorical Lyapunov theorem that characterizes stability and recovers classical stability results for discrete and continuous time systems. Importantly, classical hybrid systems admit a coalgebraic encoding, so CLT provides Lyapunov conditions for stability of hybrid systems-most notably Zeno equilibria, together with Zeno-time bounds and sufficient conditions for the existence of Zeno behavior. Finally, we outline the first steps toward open-system extensions via control coalgebras, which generalize Control Lyapunov Functions (CLFs) and Control Barrier Functions (CBFs) enabling controller synthesis.