Spaces With Jump Relations: An Approach to Hybrid Dynamics?

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Abstract

We introduce a procedure associating with a jump relation Δ defined on a proper metric space X, a topological space X_Δ with a weaker topology, such that any right-continuous and locally bounded map $c : (0, 1) \to X$ satisfying at every $s \in (0, 1)$ the condition

$$(\dagger) \quad \limsup_{t \to s-0} c(t) \subseteq \Delta^{op}(c(s)),$$

is, in fact, a continuous path in X_Δ.

A theory of hybrid dynamical systems where executions are paths of this kind would render gluing constructions unnecessary while enabling trajectories with singularities situated along the jump set. In addition to encoding the time-irreversibility of jumps in hybrid systems more adequately than in gluing-based models, this approach lets us consider more general hybrid time domains (the set of jump times need not be discrete). It also generalizes graph-based hybrid structures to more natural non-deterministic analogues (similarly to open transition systems), while offering a direct link to the theory of discrete multivalued map representations of continuous dynamics.

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